Membership Representation for Detecting Block-diagonal Structure in Low-rank or Sparse Subspace Clustering

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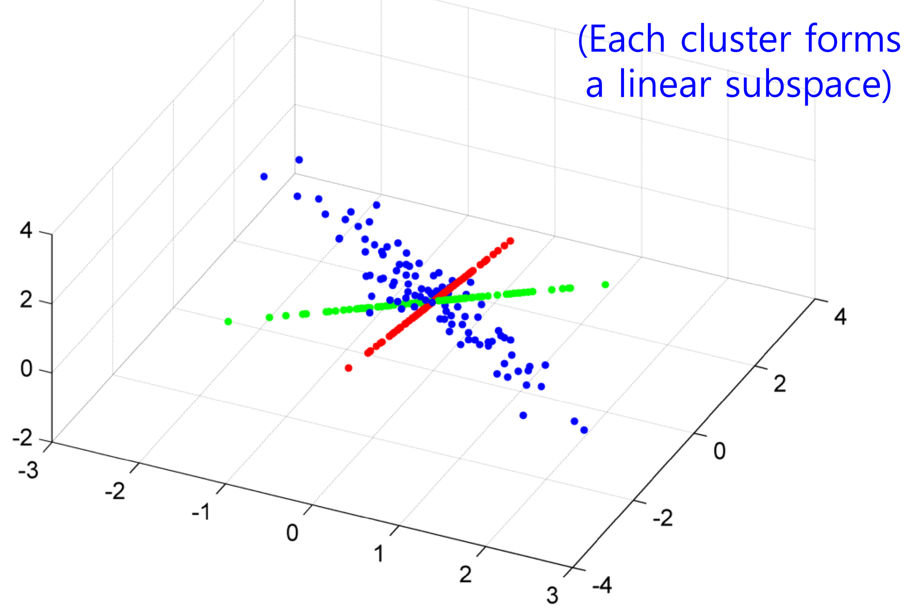
1. Overview of subspace clustering

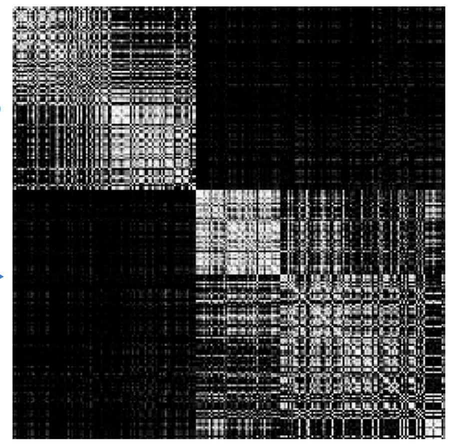
Subspace clustering divides data into groups, each of which forms a linear subspace. There are many examples that are adequate for subspace clustering, such as motion, face, and texture.



$$\min_{\mathbf{W},\mathbf{E}} \ \|\mathbf{W}\|_{\sigma_W} + \lambda_E \|\mathbf{E}\|_{\sigma_E}, \quad \text{s.t.} \quad \mathbf{X} = \mathbf{X}\mathbf{W} + \mathbf{E},$$

W: Solution matrix
E: Error matrix

LRR example



Low-Rank Representation (LRR) [Liu et al., TPAMI 2013] or Sparse Subspace Clustering (SSC) [Elhamifar and Vidal, TPAMI 2013] both solve this problem by a self expressive model with low-rank or sparsity constraint, respectively. The solution matrix is approximately block-diagonal, which requires post-processing.

3. Procedure

1. $\mathbf{W}$ is not ideal. → Minimize the errors.

2. Solutions become trivial.
→ Regularization terms

1st stage

$$\min_{\mathbf{M}} \ \|\mathbf{W}-\mathbf{W}\odot\mathbf{M}\|_1 + \lambda_M\|\mathbf{M}\|_F^2 \quad \text{s.t.} \quad \mathbf{M} \geq 0, \mathbf{M} \succeq \mathbf{0}, \operatorname{diag}(\mathbf{M}) = \mathbf{1}$$

2nd stage

$$\min_{\mathbf{F}} \ \operatorname{tr}(\mathbf{F}) \quad \text{s.t.} \quad \|\mathbf{F}-\mathbf{F}\odot\mathbf{M}\|_1 \leq c, \quad \mathbf{F} \geq 0, \mathbf{F} \succeq \mathbf{0}, \mathbf{F}\mathbf{1} = \mathbf{1}$$

3. These optimization problems are complex.
→ Auxiliary variables (e.g., $\mathbf{M}$=$\mathbf{M}_1$=$\mathbf{M}_2$)
→ Augmented Lagrangian method (ALM)

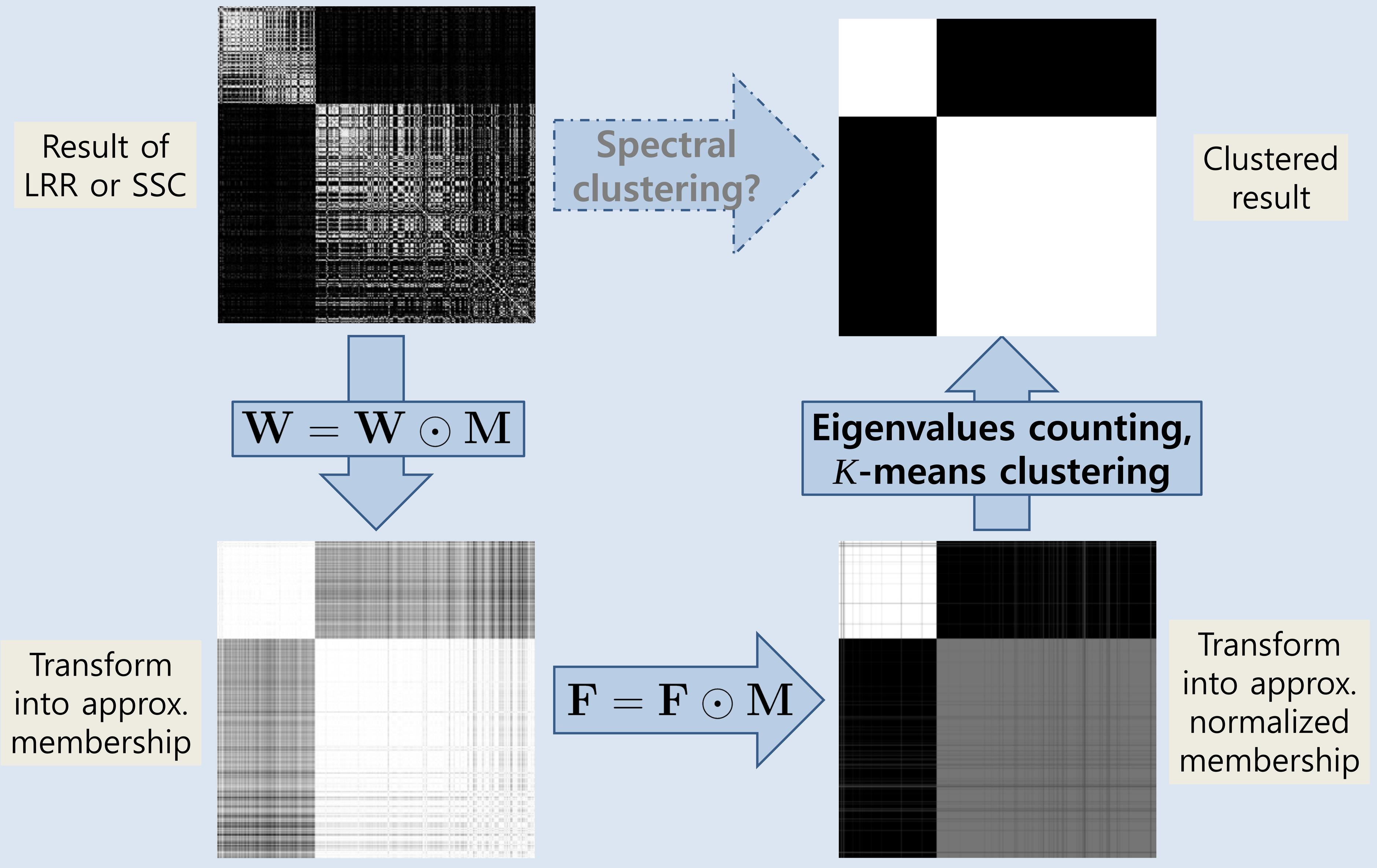
4. Sets of membership / normalized membership matrices are discrete.
→ Relax them to convex conditions.

2. Proposal: Membership representation / normalized membership representation

Properties of membership matrix

1. Each element is either zero or one. Diagonal elements are all ones.
2. It can be transformed into a block diagonal matrix, by permuting the same indices of rows and columns. Then, the blocks of the transform matrix are filled with one.
3. It is positive semidefinite (PSD).
4. It has integral eigenvalues.
5. For an ideal $\mathbf{W}$, $\mathbf{M}$ that has the same block-diagonal structure with $\mathbf{W}$ satisfy:

⇒ $\mathbf{W} = \mathbf{W}\odot\mathbf{M}$



Properties of normalized membership matrix

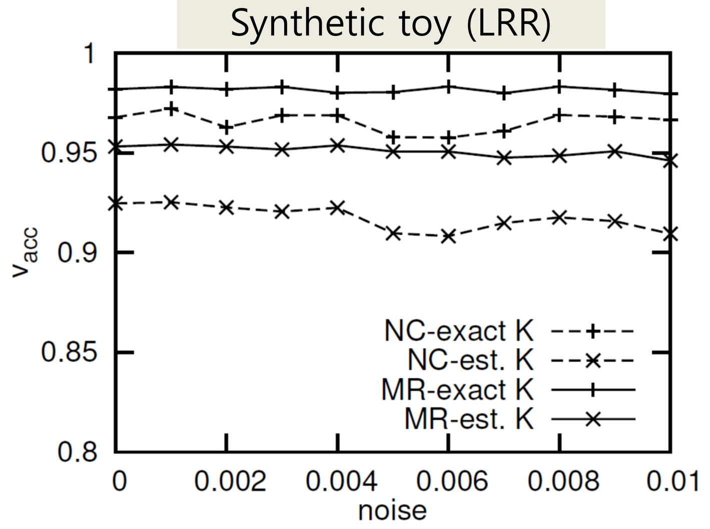
1. It is doubly stochastic, i.e., $\mathbf{F1}=\mathbf{F}^T\mathbf{1}=\mathbf{1}$.
2. It can be transformed into a block diagonal matrix, by permuting the same indices of rows and columns. Each block is filled with the reciprocal of its dimension.
3. It is an orthogonal projection, i.e., $\mathbf{F}^2=\mathbf{F}=\mathbf{F}^T$.
4. The eigenvalues are either zero or one.
5. For an ideal $\mathbf{M}$, $\mathbf{F}$ that has the same block-diagonal structure with $\mathbf{M}$ satisfy:

⇒ $\mathbf{F} = \mathbf{F}\odot\mathbf{M}$

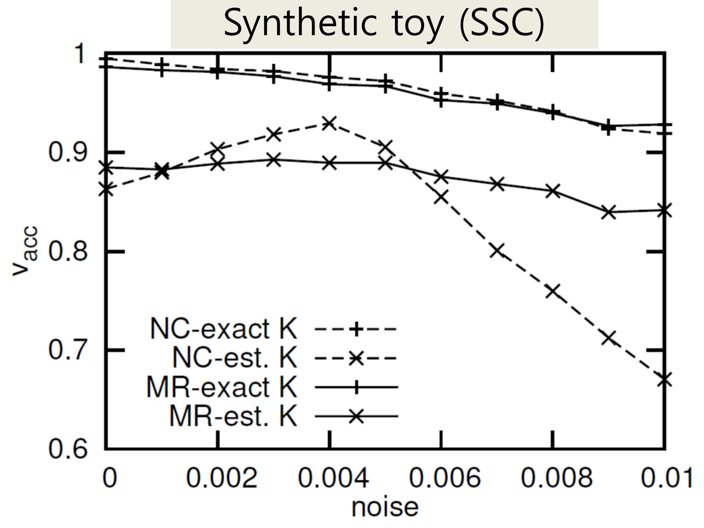
4. Results

Compared methods: Normalized cut [Shi and Malik, TPAMI 2000]

Synthetic toy (LRR)



Synthetic toy (SSC)



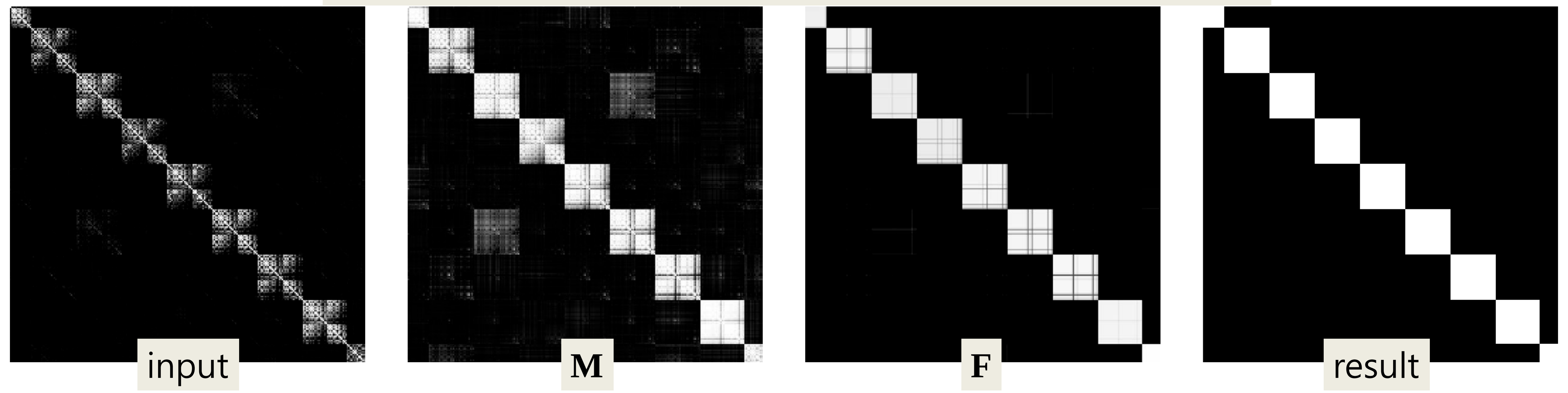
Hopkins 155

			K	v_{ACC}	v_{NMI}	p_K (%)
LRR	NC	exact		0.965 (0.077)	0.883 (0.187)	1
		est.		0.928 (0.107)	0.828 (0.236)	0.744
	MR	exact		**0.966** (0.075)	**0.891** (0.156)	0.949
		est.		**0.941** (0.103)	**0.869** (0.163)	0.814
SSC	NC	exact		**0.974** (0.073)	0.909 (0.192)	1
		est.		0.926 (0.120)	0.808 (0.320)	0.724
	MR	exact		0.970 (0.076)	**0.915** (0.158)	0.974
		est.		**0.939** (0.113)	**0.893** (0.153)	0.801

Yale-Caltech

	LRR			SSC		
	NC	MR		NC	MR	
	exact	exact	est.	exact	exact	est.
v_{ACC}	**1**	**1**	**1**	0.936	**0.982**	0.950
v_{NMI}	**1**	**1**	**1**	0.961	**0.980**	0.951
K	38	38	38	38	38	40

A part of the Yale-Caltech result (1204+α samples, 38 classes, LRR)



5. Conclusion

- We propose the membership representation (MR), which detects the block-diagonal structure in the output of subspace clustering.
- MR is a self-expressive system based on a Hadamard product.
- The final output is a normalized membership matrix, which is a doubly stochastic normalization of $\mathbf{M}$. It has eigenvalues in between zero and one.
- A simple eigenvalue-counting method showed competitive results in the experiments.

Any questions, comments, or suggestions are welcome! (Email: mleepaper@hanyang.ac.kr)