[Supplementary] Consensus of Non-Rigid Reconstructions

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I. Proof of Proposition 1

$\mathbf{H}$ can be expressed as the sum of $\mathbf{H}_k \triangleq \mathbf{W}_k \otimes \mathbf{I} - \mathbf{v}_k \mathbf{v}_k^T$ where $\mathbf{v}_k \triangleq \mathrm{vec}(\mathbf{Z}_k)/\|\mathbf{Z}_k\|$. Let $\widetilde{\mathbf{H}}_k$ be a square submatrix of $\mathbf{H}_k$ that includes rows and columns indexed by g_k. After some manipulation, we can find out that

$$\begin{aligned}\widetilde{\mathbf{H}}_k &= \mathbf{I} - \frac{1}{n_g}\mathbf{1}\mathbf{1}^T \otimes \mathbf{I} - \mathbf{v}_k|_{g_k} \mathbf{v}_k|_{g_k}^T \\ &= \mathbf{I} - \left[\tfrac{1}{\sqrt{n_g}}\mathbf{1} \otimes \mathbf{I} \quad \mathbf{v}_k|_{g_k}\right]\left[\tfrac{1}{\sqrt{n_g}}\mathbf{1} \otimes \mathbf{I} \quad \mathbf{v}_k|_{g_k}\right]^T.\end{aligned} \tag{1}$$

$\left[\tfrac{1}{\sqrt{n_g}}\mathbf{1} \otimes \mathbf{I} \quad \mathbf{v}_k|_{g_k}\right]$ turns out to be a tall orthogonal matrix, based on the following facts: (i) $\tfrac{1}{\sqrt{n_g}}\mathbf{1} \otimes \mathbf{I}$ is an orthogonal matrix. (ii) The norm of $\mathbf{v}_k|_{g_k}$ is obviously one. (iii) $\mathbf{1} \otimes \mathbf{I}$ and $\mathbf{v}_k|_{g_k}$ are mutually orthogonal, i.e.,

$$(\mathbf{1} \otimes \mathbf{I})^T \mathbf{v}_k|_{g_k} = (\mathbf{e}_k^T \otimes \mathbf{I})^T \mathbf{v}_k = \frac{\mathrm{vec}(\mathbf{Z}_k \mathbf{e}_k^T)}{\|\mathbf{Z}_k\|} = \mathbf{0}. \tag{2}$$

The last equality comes from (12) of the main document. Therefore, $\widetilde{\mathbf{H}}_k$ satisfies $\mathbf{0} \preceq \widetilde{\mathbf{H}}_k \preceq \mathbf{I}$, which is equivalent to $\mathbf{0} \preceq \mathbf{H}_k \preceq \mathrm{diag}(\mathbf{e}_k)$. By summing up these relations for all k, we have $\mathbf{0} \preceq \mathbf{H} \preceq \sum \mathrm{diag}(\mathbf{e}_k) \preceq \bar{m}_g \mathbf{I}$. This concludes the proof.

II. Performance under various parameters

The proposed weak reconstructor does not have any parameter (not even K)[1], and all the parameters are in the sampling step. For the parameters in the sampling step, we performed several experiments to check the sensitivities. As can be seen in Fig. 1, the proposed method is not sensitive to λ and m_g. It is also not sensitive to n_g when the deformation is not dynamic (drink, stretch) or the data is dense (pace). If the deformation is dynamic (capoeira, walking, yoga), errors tend to decrease as n_g decreases because a group gets more concentrated in a local region, making the reconstruction easier. For some sparse data sets (dance, walking), errors increase for very small n_g, because the local parts may contain some not-so-small deformations in the case of sparse data. In theory, n_g must be no less than four, which is the fewest number for rigid or non-rigid SfM.

[1] As explained in Section 2.2 of the main document, our weak reconstructor automatically chooses K during rotation calculation.

III. Performance under various modifications

The proposed method is composed of four steps; sampling, weak reconstructions, reflection correction, and calculation of statistics (strong reconstruction). To examine the contribution of each step to performance, we have evaluated the proposed method under various modifications. Four modifications have been tested:

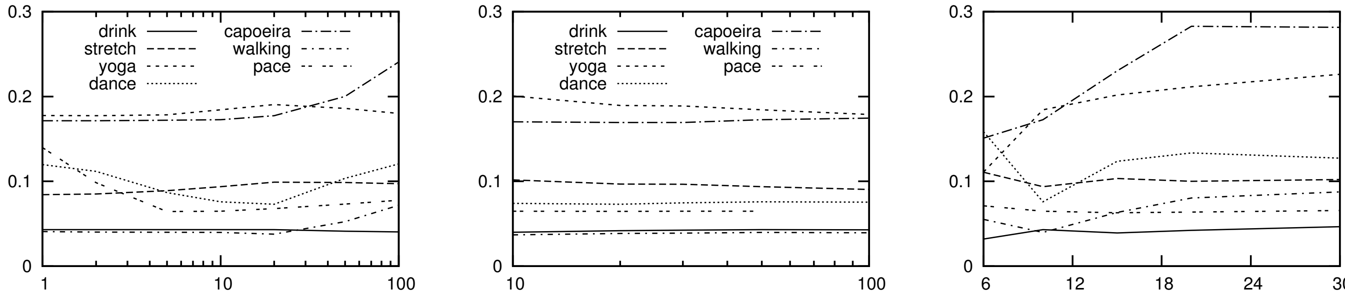
1) "All points as single group": Weak reconstructor is applied to a single group that contains the whole trajectories. This is almost the same as BMM [4], except the modifications explained in Section 2.2 of the main document.
2) "BMM as weak reconstructor": BMM is used as a weak reconstructor instead of the modified BMM in Section 2.2 of the main document.
3) "Without reflection correction": The strong reconstruction step is applied directly on weak reconstruction results, without the reflection correction step in Section 3.1 of the main document.
4) "mean instead of median (l_2 minimization)": The strong reconstruction is calculated based on mean instead of median. This corresponds to modifying the l_1-norm in (28) of the main document to an l_2-norm.

Tables I and II show the performance of these modifications. Here, using BMM as weak reconstructor generally shows worse performance than the proposed method, which is mainly due to the unstable rotation calculation step in BMM. Applying the weak reconstructor to the whole trajectories shows similar performance to BMM. We can confirm in these tables that the reflection correction step is crucial for performance, in that skipping this step gives severely degraded performance. Using mean instead of median for strong reconstruction shows worse performance than the proposed method, which confirms our argument that there can be bad or outlying weak reconstructions. These results verify that each part of the proposed algorithm has an important contribution to performance.

References

[1] L. Torresani, A. Hertzmann, and C. Bregler, "Nonrigid Structure-from-Motion: Estimating Shape and Motion with Hierarchical Priors," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 30, no. 5, pp. 878–892, May 2008. 2

[2] P. F. U. Gotardo and A. M. Martinez, "Kernel Non-Rigid Structure from Motion," in *Proc. IEEE Int'l Conf. Computer Vision*, November 2011. 2



(a) λ (b) m_g (c) n_g

Fig. 1. Average errors for various values of parameters.

TABLE I
AVERAGE RECONSTRUCTION ERRORS OF THE PROPOSED METHOD UNDER VARIOUS MODIFICATIONS (FOR THE BENCHMARK DATA SETS [1], [2]).

data set	BMM	all points as single group	BMM as weak reconstructor		without reflection correction		mean instead of median (l_2 minimization)		proposed method	
			mean	std.	mean	std.	mean	std.	mean	std.
dance	0.1454	0.1369	0.1258	0.0029	0.3468	0.0392	0.1081	0.0028	**0.0759**	0.0020
capoeira	0.2465	0.2825	0.1824	0.0010	0.2743	0.0335	0.1871	0.0023	**0.1725**	0.0010
walking	0.0862	0.0903	0.0431	0.0004	0.1655	0.0386	0.0417	0.0005	**0.0396**	0.0003
face	0.0233	**0.0177**	0.1037	0.0014	0.3540	0.0162	0.0249	0.0005	0.0248	0.0003
shark	0.1669	**0.0791**	0.1504	0.0010	0.3579	0.1157	0.0844	0.0016	0.0832	0.0000
flag	0.1741	0.0828	0.1193	0.0010	0.0638	0.0026	0.0391	0.0009	**0.0387**	0.0007
pace	0.0892	0.0845	0.0763	0.0006	0.3451	0.0129	0.0732	0.0005	**0.0648**	0.0004

TABLE II
AVERAGE RECONSTRUCTION ERRORS OF THE PROPOSED METHOD UNDER VARIOUS MODIFICATIONS (FOR THE DATA SETS IN [3] WITH REALISTIC ROTATIONS).

data set		BMM	all points as single group	BMM as weak reconstructor		without reflection correction		mean instead of median (l_2 minimization)		proposed method	
total rotation	sequence			mean	std.	mean	std.	mean	std.	mean	std.
60°	drink	**0.0411**	0.0573	0.0427	0.0013	0.1736	0.0536	0.0447	0.0003	0.0431	0.0003
	pickup	0.1580	0.1651	0.1428	0.0012	0.2112	0.0271	0.1398	0.0023	**0.1281**	0.0017
	stretch	0.0971	0.1083	0.1089	0.0014	0.2495	0.0235	0.1080	0.0044	**0.0939**	0.0016
	yoga	0.2463	0.2252	0.2164	0.0028	0.2914	0.0262	0.1962	0.0069	**0.1845**	0.0036
90°	drink	0.0919	0.0445	**0.0294**	0.0001	0.1913	0.0185	0.0362	0.0002	0.0353	0.0001
	pickup	0.1011	0.1317	0.1036	0.0022	0.1957	0.0370	0.0963	0.0014	**0.0918**	0.0023
	stretch	0.0773	0.3006	**0.0619**	0.0008	0.1893	0.0340	0.0826	0.0025	0.0797	0.0013
	yoga	0.1686	0.1583	0.1953	0.0029	0.2748	0.0323	0.1549	0.0035	**0.1190**	0.0026
120°	drink	0.0424	0.0391	0.0315	0.0001	0.1487	0.0185	0.0337	0.0004	**0.0304**	0.0001
	pickup	0.1104	0.1220	0.1104	0.0010	0.1880	0.0236	0.1107	0.0017	**0.0964**	0.0012
	stretch	0.0930	0.0917	0.0976	0.0010	0.2368	0.0236	0.0955	0.0012	**0.0846**	0.0012
	yoga	0.1309	0.1840	0.1248	0.0034	0.1773	0.0347	0.1447	0.0024	**0.1115**	0.0024

[3] I. Akhter, Y. Seikh, S. Khan, and T. Kanade, "Trajectory Space: A Dual Representation for Nonrigid Structure from Motion," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 33, no. 7, pp. 1442–1456, July 2011. 2

[4] Y. Dai, H. Li, and M. He, "A Simple Prior-free Method for Non-Rigid Structure-from-Motion Factorization," in *Proc. IEEE Conf. Computer Vision and Pattern Recognition*, June 2012. 1