A Procrustean Markov Process for Non-Rigid Structure Recovery

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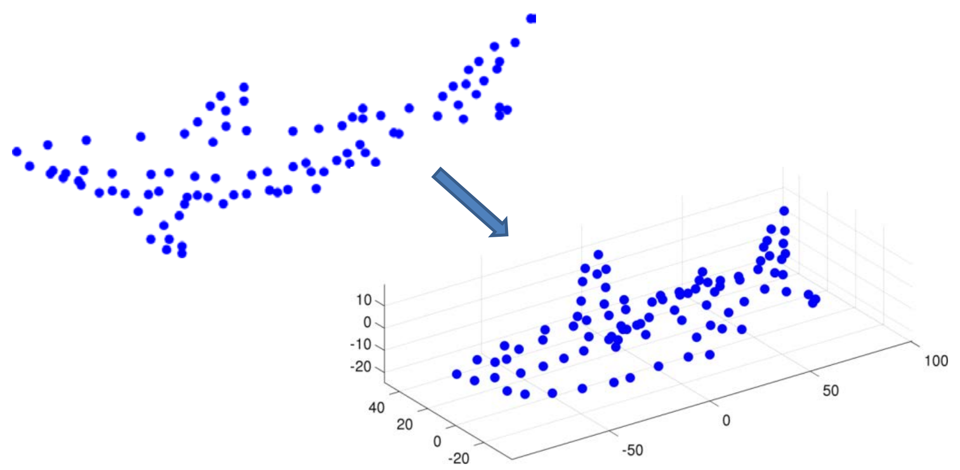
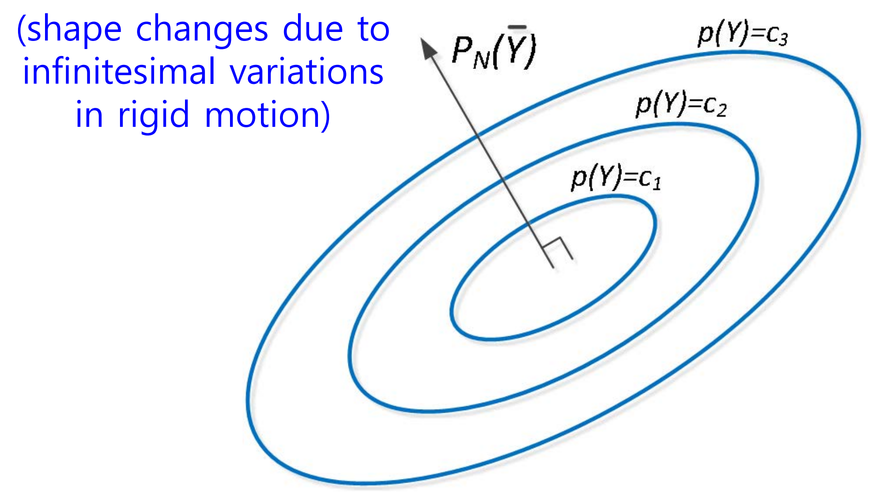
1. Previous work: Procrustean Normal Distribution

A Procrustean normal distribution (PND) [Lee et al., CVPR 2013] is a probability distribution of Procrustes-aligned shapes. It was designed for solving non-rigid structure from motion, which is a problem that reconstructs non-rigid 3D structures from a set of 2D observations.





$$\mathbf{Y} \sim \mathcal{N}_P\left(\bar{\mathbf{Y}}, \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^T\right)$$

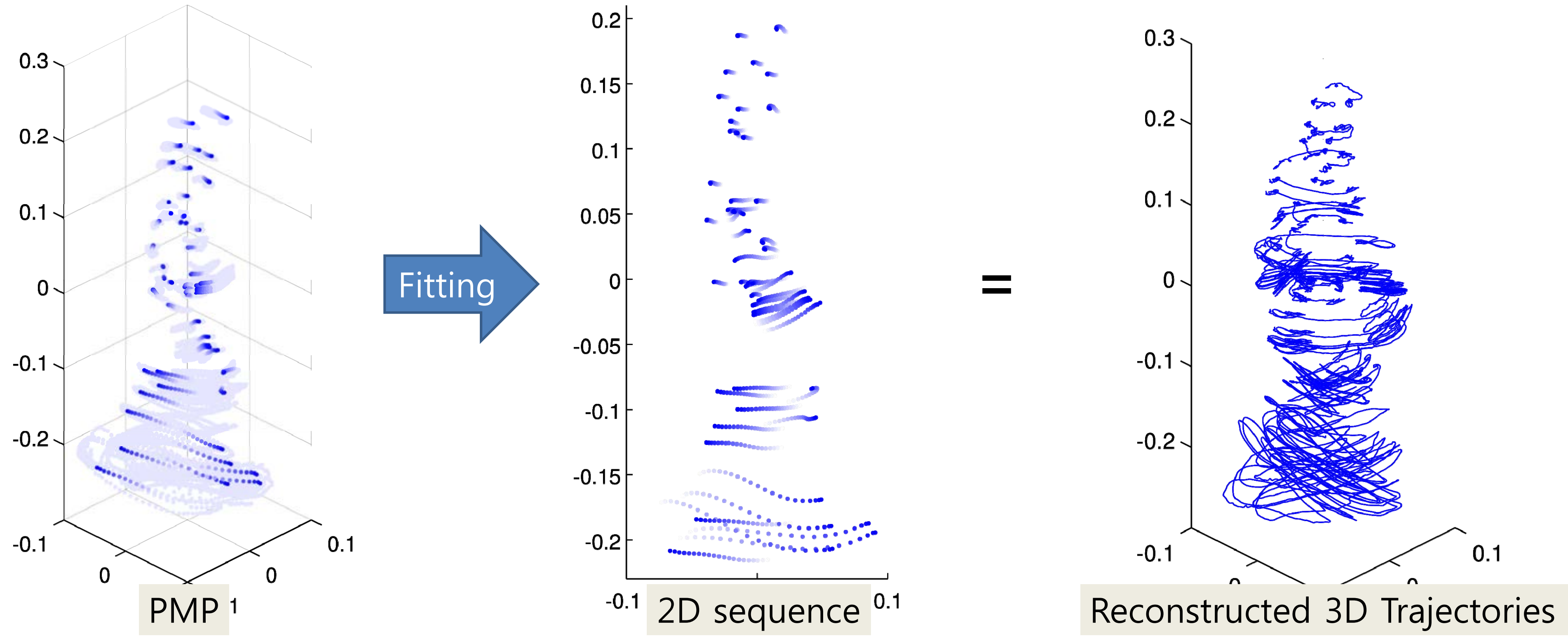
A PND excludes any shape changes due to rigid motions to strictly separate rigid/non-rigid shape variations, which leads to a better reconstruction performance.

3. EM-PMP

Fit a PMP to a sequence of 2D observation.

E-step is efficiently performed by the Kalman smoothing procedure.

In **M-step**, the parameters including the smoothness parameter are updated.



2. Procrustean Markov Process: A Markov process of Procrustes-aligned shapes

Properties of PMP

1. A first-order Markov Process.

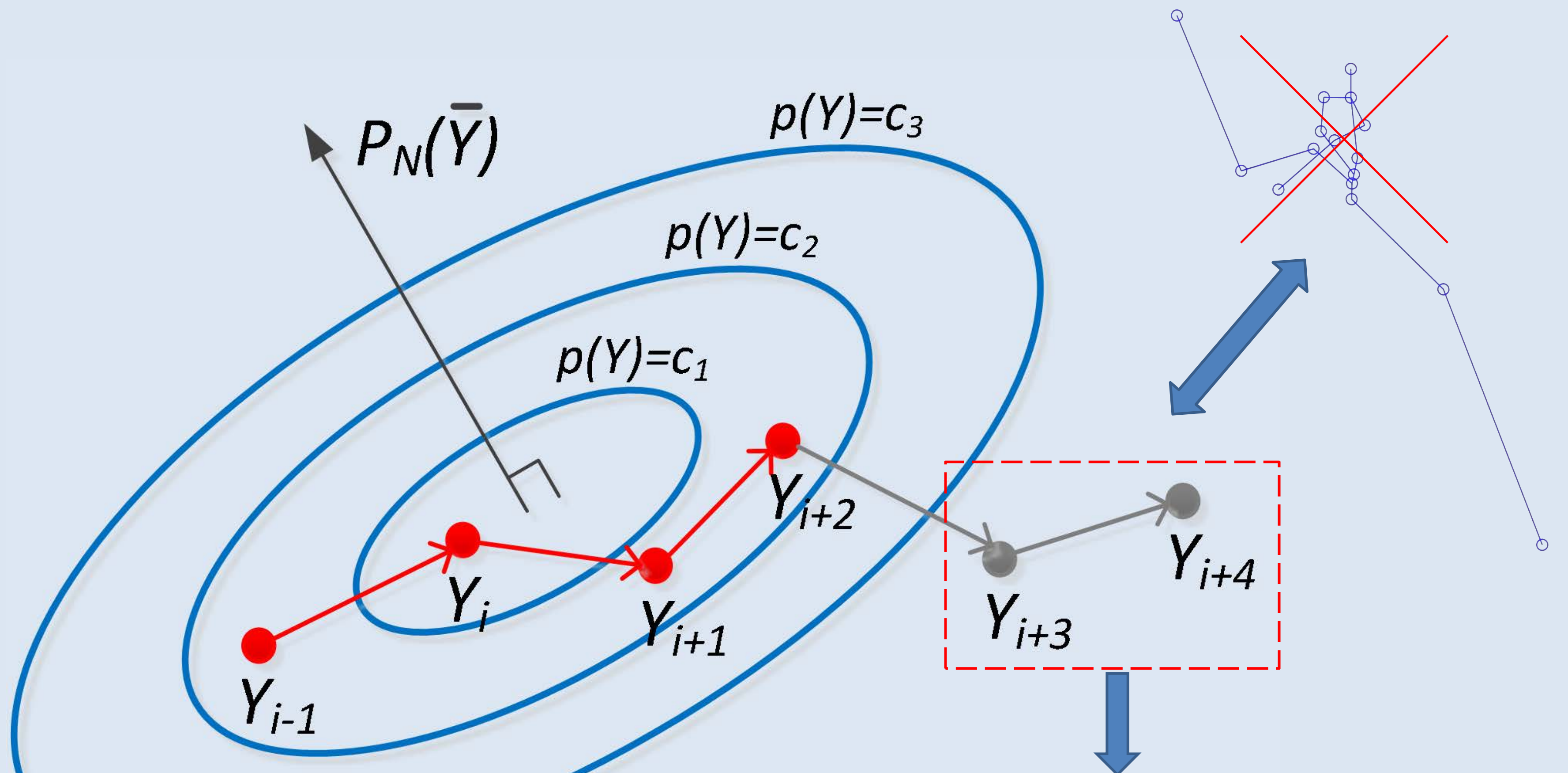
$$\mathrm{vec}(\mathbf{Y}_i) = \alpha\,\mathrm{vec}\left(\mathbf{Y}_{i-1} - \bar{\mathbf{Y}}\right) + \mathrm{vec}\left(\bar{\mathbf{Y}}\right) + \boldsymbol{\omega}_i,$$

$$\boldsymbol{\omega}_i \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}\mathbf{H}\mathbf{Q}^T\right)$$

2. Each state of PMP satisfies the PND constraint, i.e., it is aligned to the mean shape by the Procrustes principle.

$$P\left(\mathbf{P}_N\left(\bar{\mathbf{Y}}\right)^T \mathrm{vec}(\mathbf{Y}_i) = 0\right) = 1$$

3. The steady-state distribution is a PND.



This kind of changes are not likely in shape deformation, because deformation is usually finite and recurrent.

$\{Y_i\}$: PMP sequence
$\bar{Y}$: mean shape of steady-state dist.

Stationarity constraint in PMP

To rule out such unlikely shapes from the shape distribution, we assume that PMP is a stationary process.

1. Hence, the marginal distribution of each shape is the same as the steady-state distribution, i.e., PND.

$$\mathbf{Y}_i \sim \mathcal{N}_P\left(\bar{\mathbf{Y}}, \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^T\right)$$

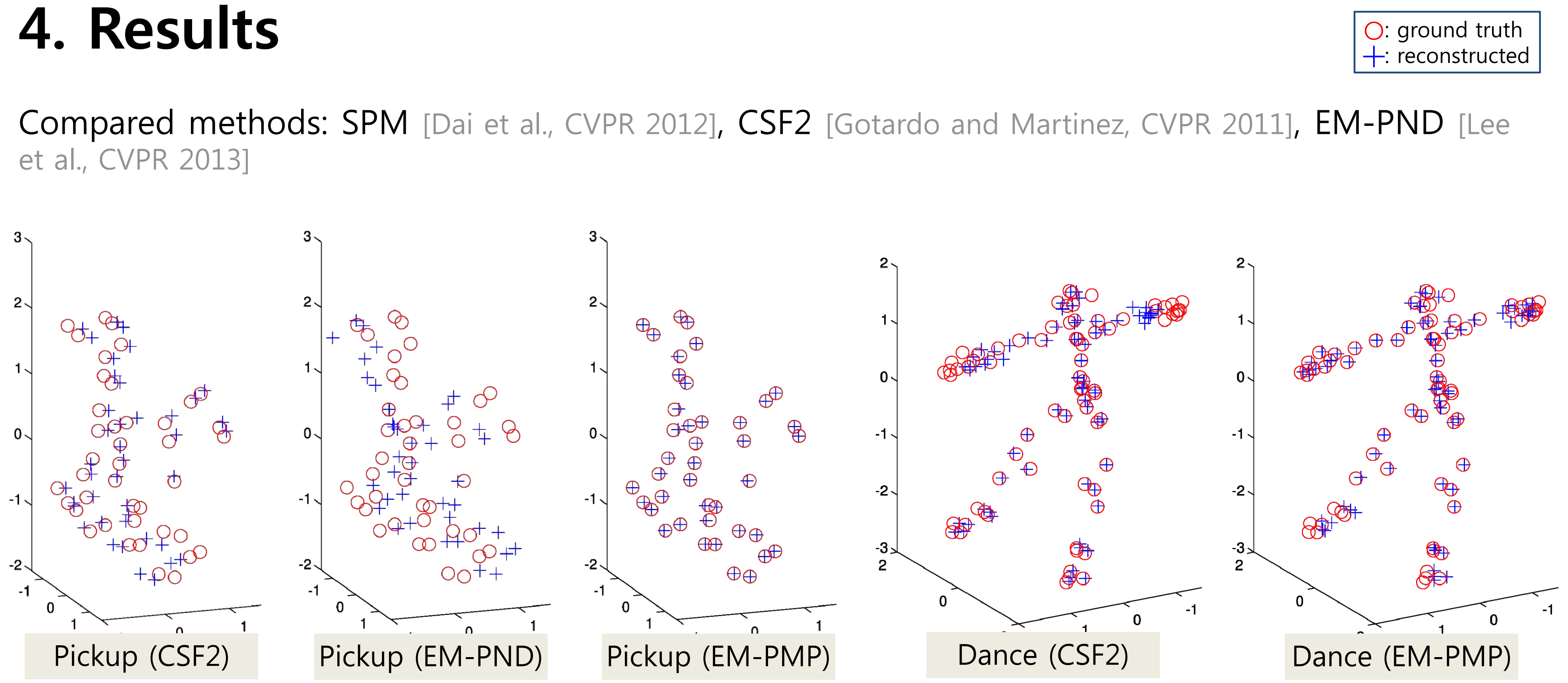
2. This condition is ensured by imposing the Lyapunov condition.

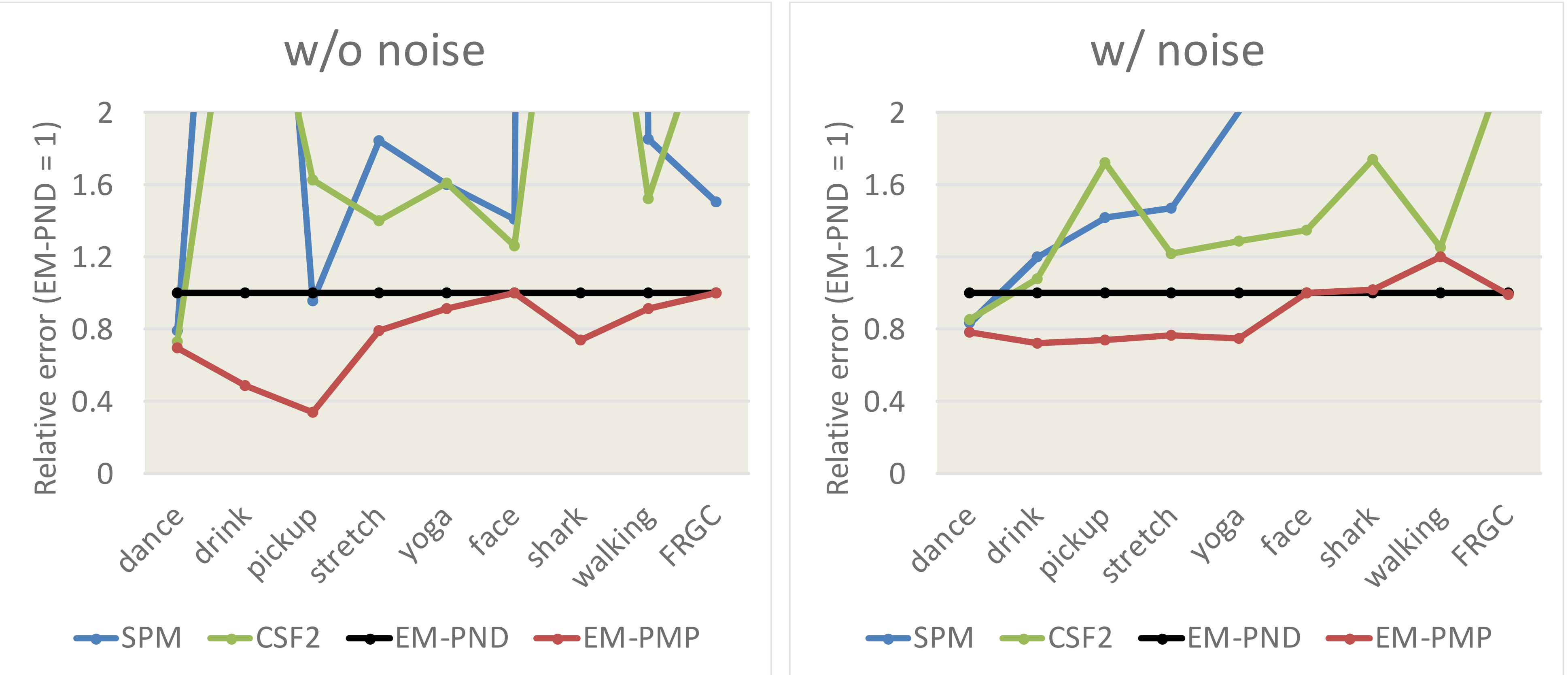
$$\mathbf{\Sigma} = \alpha^2\mathbf{\Sigma} + \mathbf{H}$$

3. PMP is a reversible Markov process. (↔ It satisfies detailed balance.)

4. Results

Compared methods: SPM [Dai et al., CVPR 2012], CSF2 [Gotardo and Martinez, CVPR 2011], EM-PND [Lee et al., CVPR 2013]





5. Conclusion

- We propose the PMP, which is an extended version of PND for temporal dependence.
- Marginal distribution of a state of a PMP is a PND.
- Shape deformation is finite, so defining PMP to be stationary improves the performance
- Absolutely no prior information about the data is required for EM-PMP.
- Proposed method gives the state-of-the-art performance, about 25% better than EM-PND.

Any questions, comments or suggestions are welcome! (Email: mlee.paper@gmail.com)