[Supplementary] A Procrustean Markov Process for Non-Rigid Structure Recovery

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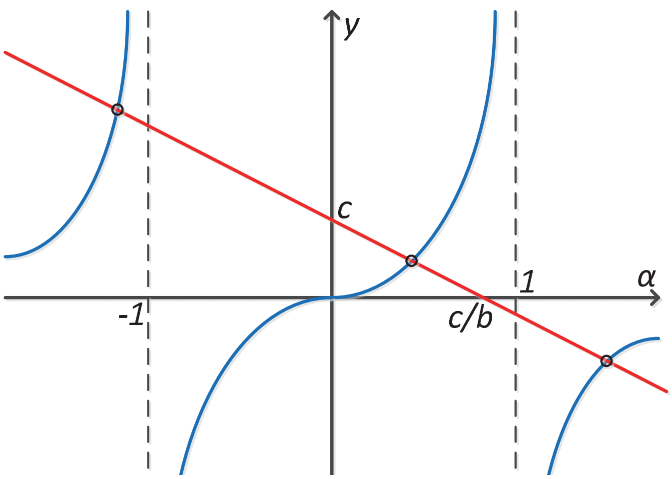

Fig. 1. Plots of $y = c - b\alpha$ and $y = (3n_p - 7)\frac{\alpha}{1-\alpha^2}$.

I. Proof of the Uniqueness of α

Here, we prove the uniqueness of α in M-step of EM-PMP. The equation $\partial J / \partial \alpha = 0$ can be rearranged as

$$c - b\alpha = (3n_p - 7)\frac{\alpha}{1-\alpha^2}, \tag{1}$$

where

$$\begin{aligned} b &= \sum_{i=2}^{n_s - 1} \mathrm{tr}\big(\mathbf{Q}\mathbf{H}^{-1}\mathbf{Q}^T \left(\mathbf{h}_i \mathbf{h}_i^T + \mathbf{C}_i\right)\big) > 0, \\ c &= \sum_{i=2}^{n_s} \mathrm{tr}\big(\mathbf{Q}\mathbf{H}^{-1}\mathbf{Q}^T \left(\mathbf{h}_{i-1} \mathbf{h}_i^T + \mathbf{C}_{i-1,i}\right)\big). \end{aligned} \tag{2}$$

To see the characteristics of this equation, the plots of $y = c - b\alpha$ and $y = (3n_p - 7)\frac{\alpha}{1-\alpha^2}$ are shown in Fig. 1. Note that $y = c - b\alpha$ is always decreasing because $b > 0$, and $y = (3n_p - 7)\frac{\alpha}{1-\alpha^2}$ is increasing from $-\infty$ to ∞ in the range of $[-1, 1]$. Therefore, there is always one real solution in the range of $[-1, 1]$. Since the valid range of α is $[-1, 1]$, we can always find a unique solution α^* that minimizes J.

II. Experiments on a Large Number of Landmarks

We have performed experiments on the data with a large number of landmarks, such as the MOCAP sequences of pants in [1]. EM-PMP, EM-PND, and CSF2[1] have been tested on 727 landmarks from the "jump" sequence [1] (291 frames). The average reconstruction errors of EM-PMP, EM-PND, and CSF2 were 0.1035, 0.1066, and 0.1334, respectively. We also performed experiments with missing data that are spatially and temporally correlated. We have artificially created such a case by setting about 50% of the points as missing, based on the relative location of each point from the camera. In this case, the average errors of EM-PMP, EM-PND, and CSF2 were 0.1435, 0.1449, and 0.1986, respectively. The Procrustean-type algorithms give better performances in these cases as expected. The videos for these cases are also provided as supplementary materials.

[1] We only tested these schemes because they showed better performance than the others in most of our experiments.

References

[1] R. White, K. Crane, and D. Forsyth, "Capturing and animating occluded cloth," in *ACM Trans. Graphics (SIGGRAPH)*, August 2007.