Procrustean Normal Distribution for Non-Rigid Structure from Motion

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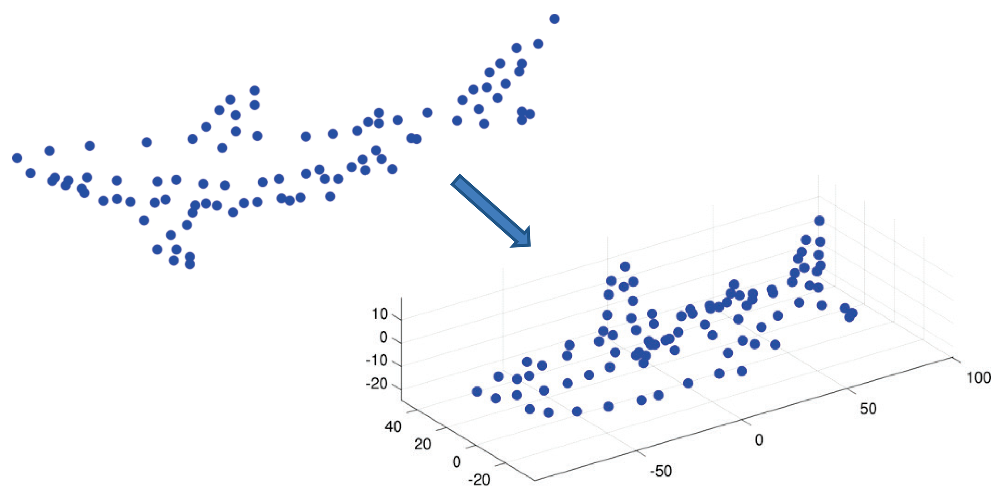
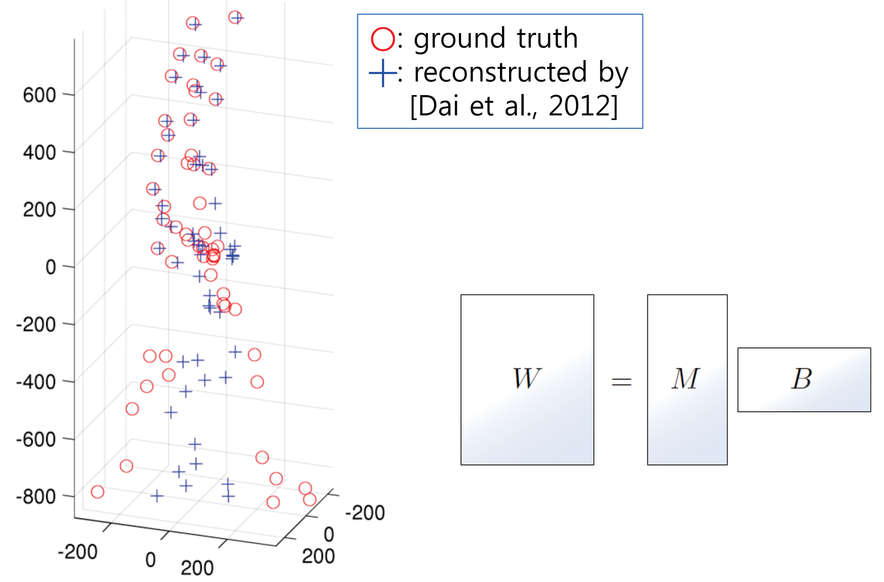
1. Overview

Non-Rigid Structure from motion (NRSfM) is a problem to estimate 3D shapes and pose of a deforming object from a set of 2D observations. Our aim is to provide accurate solutions for this problem, by incorporating more robust constraints on rigid motions.





Existing methods (based on factorization) fails to achieve good performance because the rotation estimates are inaccurate. This is originated from using algebraic error measure in finding rotations. Algebraic measure can be vulnerable to noise and ill-chosen parameters.

2. Proposal : Procrustean Normal Distribution (PND)

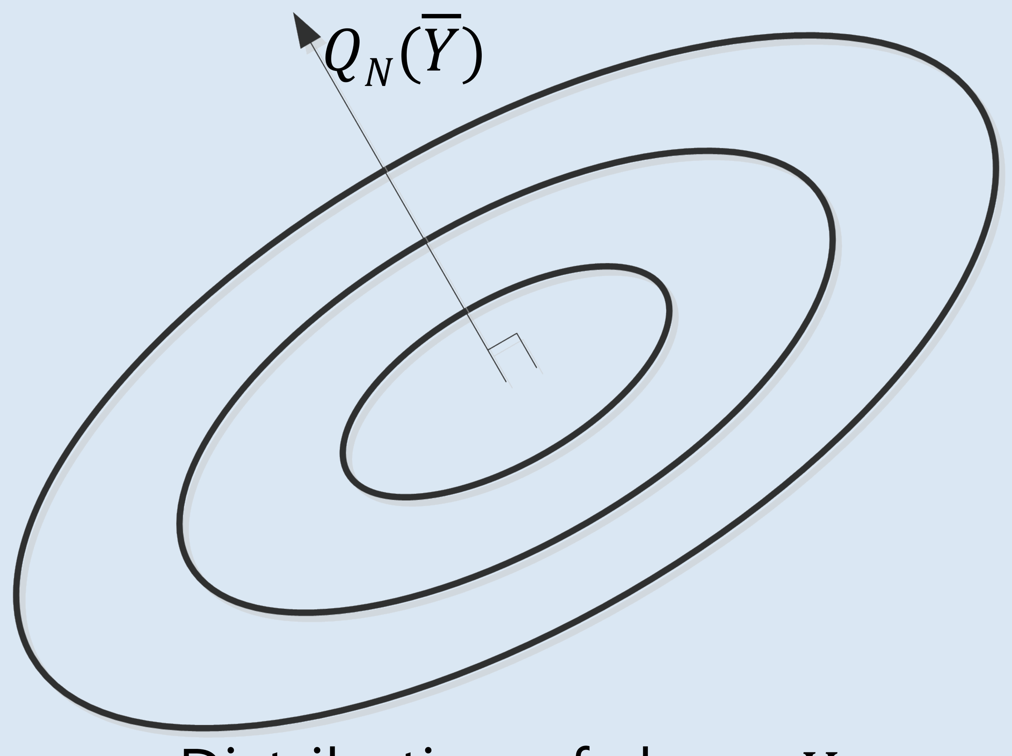
→A normal distribution with the modified GPA constraint

$$\mathrm{vec}(\mathbf{Y}) = \mathbf{Q}\mathbf{u} + \mathrm{vec}(\overline{\mathbf{Y}}),$$
$$\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{u}}), \quad \mathbf{Q}^T\mathbf{Q}_N = \mathbf{0}.$$

➡ $$\mathbf{Y} \sim \mathcal{N}_P(\overline{\mathbf{Y}}, \boldsymbol{\Sigma})$$

Properties of PND

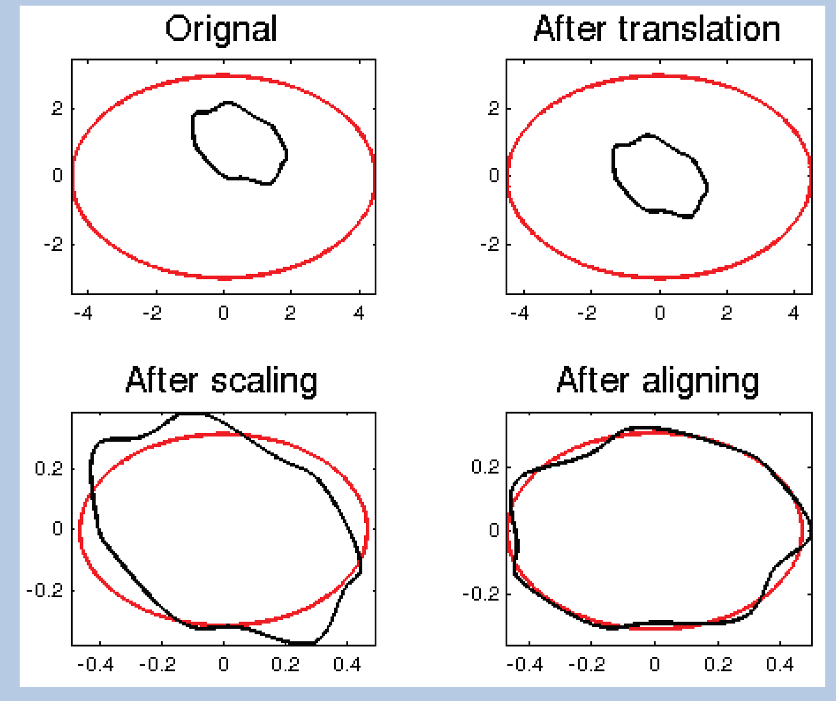
1. A general model for deformable objects.
2. Does not include any rigid motions in the model.
3. No rank restriction on deformation.
4. Rotation of a PND is also a PND.



Distribution of shape Y

PND is a shape distribution that is orthogonal to the similarity transform of the mean shape. It represents the distribution of shape deformation.

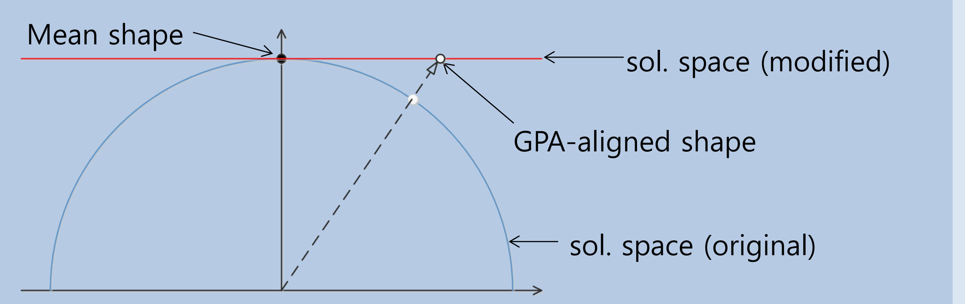
Generalized Procrustes Analysis (GPA) is the most basic form of rigid alignment. Since it is based on Gaussian noise assumption, it might find rigid motions more robustly.



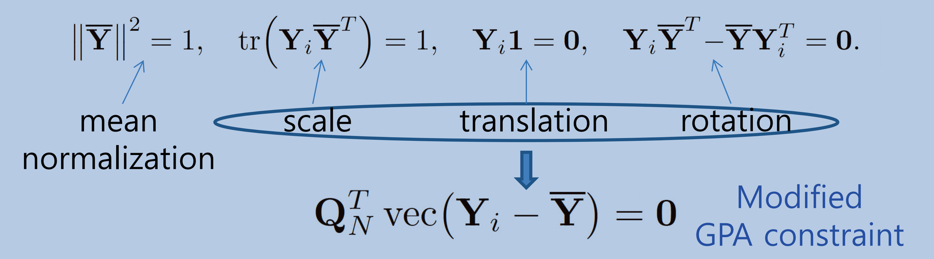
[Townsend, 2011]

Some adjustments on GPA

1. We change the scale constraint of GPA to make the solution space linear.



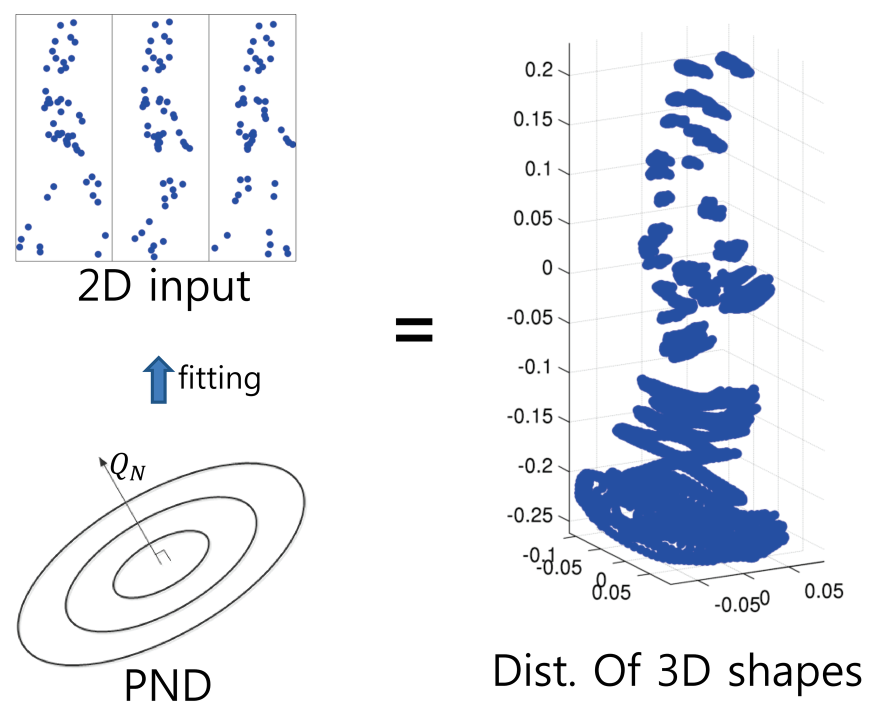
2. We derive the modified GPA constraint from the optimality condition.



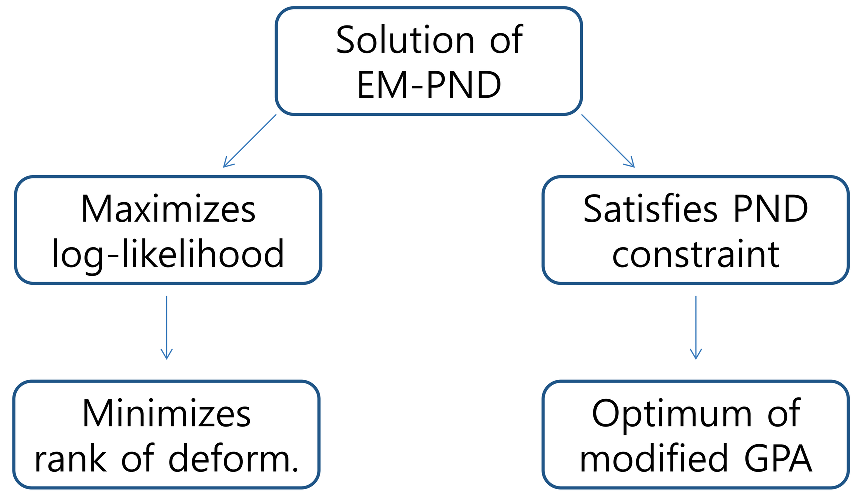
Q_N is a function of the mean shape $\overline{Y}$, which defines the null space of deformation. It is closely related to the similarity transform.

3. EM-PND

NRSfM now becomes a problem that fits PND to given 2D data.



We solve this problem using EM formulation. The cost function is highly complex, but in fact, it converges quite fast (within 0.5~2 min).



→ EM-PND chooses GPA-aligned shapes, that minimizes the rank of deformation.

4. Results

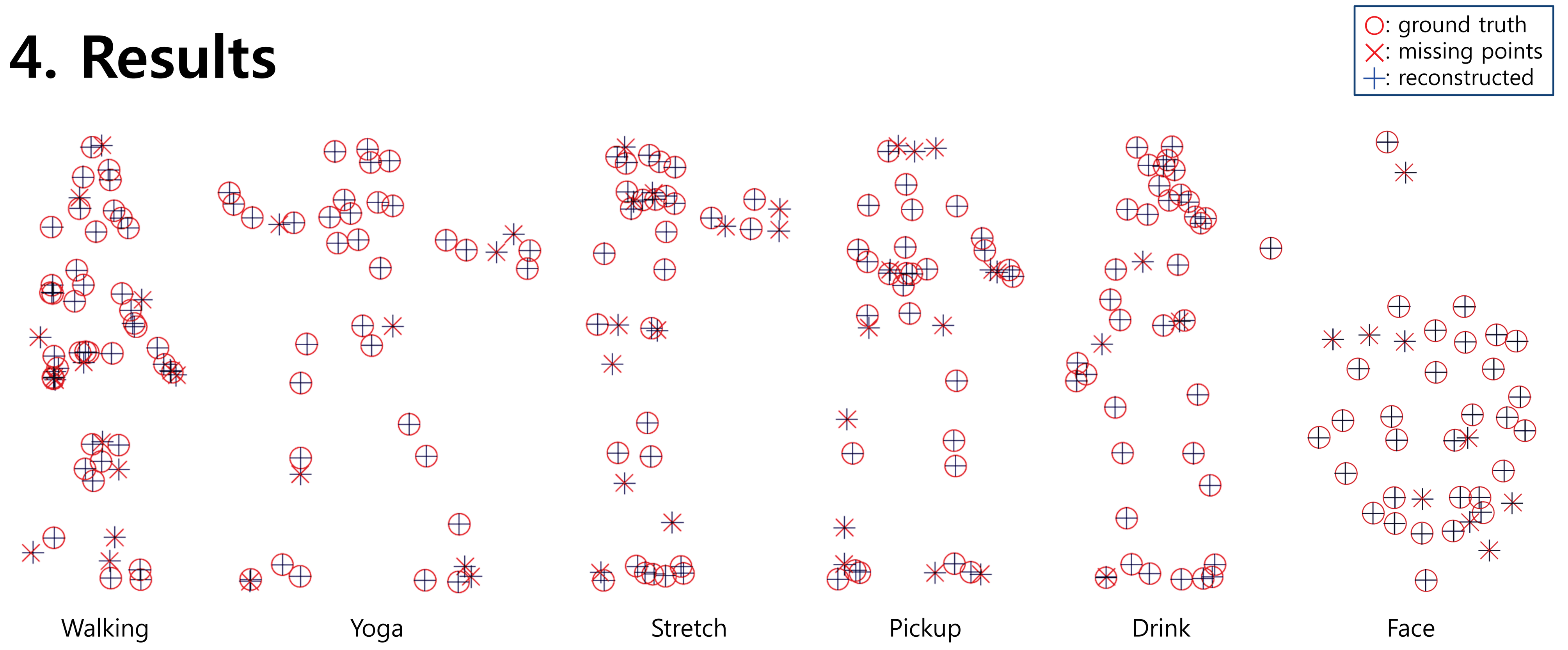


Table 1. Average reconstruction errors without noise and missing data

data \ method	EM-PPCA	MP	CSF2	SPM	EM-PND
FRGC	0.1469	0.1395	0.1926	0.1094	**0.0727**
walking	0.1485	0.2699	0.0708	0.0861	**0.0465**
shark	0.0688	0.0874	0.0551	0.5475	**0.0135**
face	0.0208	0.0329	0.0209	0.0233	**0.0165**
yoga	0.61	0.632	0.0226	0.0224	**0.014**
stretch	0.5393	0.5792	0.0219	0.0288	**0.0156**
pickup	0.5149	0.3465	0.0607	**0.0356**	0.0372
drink	0.1292	0.2707	0.0123	0.0216	**0.0037**
dance	0.2325	0.4838	**0.1349**	0.1454	0.1834

Table 2. Average reconstruction errors with noise and without missing data

data \ method	EM-PPCA	MP	CSF2	SPM	EM-PND
FRGC	0.198	0.1408	0.2061	0.184	**0.0889**
walking	0.1364	0.321	0.0966	–	**0.077**
shark	**0.0486**	0.118	0.1043	0.1784	0.06
face	0.0464	0.0523	0.0543	0.1054	**0.0403**
yoga	0.5287	0.6318	0.0529	0.0822	**0.0409**
stretch	0.5479	0.5806	0.0543	0.0652	**0.0444**
pickup	0.5037	0.3695	0.0705	0.0581	**0.0409**
drink	0.1764	0.2719	0.0365	0.0407	**0.0339**
dance	0.2229	0.4354	0.1544	**0.151**	0.1806

Table 3. Average reconstruction errors with missing data and without noise

data \ method	EM-PPCA	MP	CSF2	EM-PND
FRGC	0.1764	0.1431	0.4505	**0.0805**
walking	0.1361	0.2819	0.1033	**0.0469**
shark	0.1374	0.1077	0.0653	**0.0166**
face	0.2978	0.0456	0.0412	**0.0177**
yoga	0.1463	0.5768	0.0854	**0.0181**
stretch	0.6735	0.6149	0.0597	**0.015**
pickup	0.4969	0.4053	0.0933	**0.0149**
drink	0.1692	0.2612	0.0357	**0.0055**
dance	0.2632	0.3951	**0.1415**	0.1766

Table 4. Average reconstruction errors with noise and missing data

data \ method	EM-PPCA	MP	CSF2	EM-PND
FRGC	0.2196	0.1469	0.4574	**0.0968**
walking	0.1541	0.2853	0.1095	**0.0842**
shark	0.1323	0.1101	0.0872	**0.0672**
face	0.2273	0.0666	0.0583	**0.0464**
yoga	0.1401	0.5765	0.1583	**0.0488**
stretch	0.6881	0.6211	0.0707	**0.0535**
pickup	0.4931	0.401	0.0948	**0.0486**
drink	0.1868	0.2644	0.0428	**0.0408**
dance	0.2732	0.3986	**0.1501**	0.1601

5. Conclusion

- Finding rigid motions robustly is very important in solving NRSfM.
- We proposed PND, which is a general model for deformable objects.
- EM-PND solves NRSfM by fitting PND to given 2D data.
- Our proposal outperforms the state-of-the-art schemes, and converges fast.
- This approach can also be extended to other cases: Outlier, temporal dependence, etc.

Please send any comments or questions to my email: mlee.paper@gmail.com