[Supplementary] Procrustean Normal Distribution for Non-Rigid Structure from Motion

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TABLE I
AVERAGE RECONSTRUCTION ERRORS WITHOUT NOISE AND MISSING DATA

	CSF2	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.1926	0.0727	0.0727	0.0695	0.0695
walking	0.0708	0.0465	0.1632	0.0455	0.1274
shark	0.0551	0.0135	0.0156	0.0204	0.0925
face	0.0209	0.0165	0.0247	0.0197	0.0827
yoga	0.0226	0.014	0.2013	0.028	0.4157
stretch	0.0219	0.0156	0.3472	0.0151	0.0152
pickup	0.0607	0.0372	0.0341	0.1505	0.1513
drink	0.0123	0.0037	0.0036	0.0036	0.0035
dance	0.1349	0.1834	0.2199	0.203	0.2104

TABLE II
AVERAGE RECONSTRUCTION ERRORS WITH NOISE AND WITHOUT MISSING DATA

	CSF2	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.2061	0.0889	0.0889	0.0859	0.0859
walking	0.0966	0.077	0.1128	0.0793	0.1292
shark	0.1043	0.06	0.073	0.0524	0.0612
face	0.0543	0.0403	0.0403	0.0384	0.0382
yoga	0.0529	0.0409	0.0687	0.0395	0.4078
stretch	0.0543	0.0444	0.1035	0.0443	0.1024
pickup	0.0705	0.0409	0.0462	0.2233	0.2452
drink	0.0365	0.0339	0.0339	0.0339	0.0339
dance	0.1544	0.1806	0.1975	0.165	0.3673

TABLE III
AVERAGE RECONSTRUCTION ERRORS WITH MISSING DATA AND WITHOUT NOISE

	CSF2	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.4505	0.0805	0.0806	0.0773	0.0773
walking	0.1033	0.0469	0.0847	0.0507	0.0787
shark	0.0653	0.0166	0.0163	0.0166	0.0214
face	0.0412	0.0177	0.0167	0.018	0.0289
yoga	0.0854	0.0181	0.0732	0.0225	0.0925
stretch	0.0597	0.015	0.3796	0.0147	0.0338
pickup	0.0933	0.0149	0.1741	0.3271	0.4047
drink	0.0357	0.0055	0.0055	0.0054	0.0054
dance	0.1415	0.1766	0.1912	0.2142	0.2509

TABLE IV
AVERAGE RECONSTRUCTION ERRORS WITH NOISE AND MISSING DATA

	CSF2	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.4574	0.0968	0.0968	0.0933	0.0934
walking	0.1095	0.0842	0.1156	0.0866	0.1112
shark	0.0872	0.0672	0.0766	0.0526	0.0635
face	0.0583	0.0464	0.0745	0.0435	0.0433
yoga	0.1583	0.0488	0.094	0.0528	0.0613
stretch	0.0707	0.0535	0.299	0.0534	0.0848
pickup	0.0948	0.0486	0.0485	0.2906	0.3387
drink	0.0428	0.0408	0.0411	0.0406	0.0407
dance	0.1501	0.1601	0.1532	0.2084	0.27

I. PERFORMANCE UNDER VARIOUS MODIFICATIONS

One of the reviewers asked about how much the rotation initialization [1] and the scale constraint in EM-PND affect the performance. We show the comparison in Tables I – IV. Here, the performance of EM-PND with random initial rotations is significantly degraded for 40% of the examples. These correspond to the data with large deformations, as stated in the paper. However, when EM-PND uses the same rotation initialization method as CSF2 [1], EM-PND achieves up to 85% better performance than CSF2. This means that EM-PND contributes significantly to the performance improvement.

As discussed in the paper, a scale constraint is required for EM-PND to normalize the scale changes due to camera motions. To verify this, we performed experiments on EM-PND without the scale constraint. In Tables I – IV, the performance decreased significantly without the scale constraint for 20% of the examples. To confirm our claim, we also added some scale changes to the data. We linearly increased the frame scale as the frame number increased until the scale of the middle frame

became two, and then reverse the process so that the last frame had the same size as the original frame. The results for these data are shown in Tables V – VIII. Here, we can see that the performance of EM-PND with no scale constraint gets even worse for the data under scale change, unlike the original EM-PND.

II. DOF OF PND

The DOF of PND is $n_Y - n_d - 1$ for $\overline{\mathbf{Y}}$ and $n_R(n_R+1)/2$ for $\boldsymbol{\Sigma}$. This is derived from (14). Since $\|\overline{\mathbf{Y}}\| = 1$ and $\overline{\mathbf{Y}}\mathbf{1} = \mathbf{0}$, these relations rule out $n_d + 1$ DOF from $\overline{\mathbf{Y}}$. After $\overline{\mathbf{Y}}$ is decided, $\mathbf{Q}_N$ and $\mathbf{Q}$ are determined based on $\overline{\mathbf{Y}}$. Hence, the only term that can change in $\boldsymbol{\Sigma}$ is $\boldsymbol{\Sigma}_R$, which is a symmetric matrix. Therefore, the DOF of $\boldsymbol{\Sigma}$ is $n_R(n_R+1)/2$.

REFERENCES

[1] P. F. U. Gotardo and A. M. Martinez, "Non-rigid structure from motion with complementary rank-3 spaces," in *Proc. IEEE Conf. Computer Vision and Pattern Recognition*, June 2011. 1

TABLE V

AVERAGE RECONSTRUCTION ERRORS WITHOUT NOISE AND MISSING DATA
UNDER SCALE CHANGE

	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.073	0.073	0.0718	0.0719
walking	0.0467	0.0814	0.2006	0.1156
shark	0.0139	0.0137	0.0941	0.1099
face	0.0164	0.026	0.0345	0.1326
yoga	0.0143	0.2284	0.05	0.5727
stretch	0.0161	0.5372	0.0251	0.3372
pickup	0.0366	0.0429	0.1288	0.0451
drink	0.0035	0.0035	0.0038	0.0038
dance	0.183	0.2255	0.4783	1.2941

TABLE VI

AVERAGE RECONSTRUCTION ERRORS WITH NOISE AND WITHOUT
MISSING DATA UNDER SCALE CHANGE

	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.0896	0.0896	0.0883	0.0883
walking	0.0781	0.1059	0.1119	0.1552
shark	0.0628	0.0937	0.0983	0.1304
face	0.0428	0.0427	0.0424	0.0423
yoga	0.0436	0.109	0.0451	0.3502
stretch	0.0451	0.1159	0.0467	0.2624
pickup	0.0422	0.0874	0.0427	0.1517
drink	0.0349	0.0349	0.0339	0.034
dance	0.1865	0.1906	0.2138	0.4299

TABLE VII

AVERAGE RECONSTRUCTION ERRORS WITH MISSING DATA AND WITHOUT
NOISE UNDER SCALE CHANGE

	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.0806	0.0806	0.0794	0.0794
walking	0.0472	0.0899	0.1066	0.1016
shark	0.0161	0.0159	0.2806	0.0973
face	0.0176	0.0169	0.0216	0.0249
yoga	0.0187	0.046	0.0306	0.1988
stretch	0.0152	0.2227	0.1577	0.1041
pickup	0.015	0.1494	0.2093	0.6444
drink	0.0053	0.0054	0.0066	0.0061
dance	0.1778	0.187	0.3985	0.3717

TABLE VIII

AVERAGE RECONSTRUCTION ERRORS WITH NOISE AND MISSING DATA
UNDER SCALE CHANGE

	EM-PND	EM-PND/ random initial rotation	EM-PND/ without scale constraint	EM-PND/ random initial +no constraint
FRGC	0.0972	0.0973	0.0958	0.0959
walking	0.0873	0.1091	0.1142	0.13
shark	0.0763	0.0801	0.1079	0.0893
face	0.0488	0.0488	0.0485	0.0484
yoga	0.0548	0.1079	0.0586	0.1002
stretch	0.0539	0.2159	0.0552	0.1709
pickup	0.0499	0.0499	0.0502	0.6887
drink	0.042	0.0424	0.0409	0.041
dance	0.1659	0.165	0.2557	0.3012